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Dual-Phase-Lag Model on Thermoelastic Interactions in a Semi-Infinite Medium Subjected to a Ramp-Type Heating

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A finite element method is used to study the dual-phase-lag model on thermoelastic interactions in a semi-infinite medium subjected to a ramp-type heating. The governing equations are taken in a unified system from which the field equations for coupled thermoelasticity as well as for Lord and Shulman theory can be easily obtained as particular cases. Due attention has been paid to the finite time of rise of temperature, displacement and stress. The finite element method is proposed to analyze the problem and obtain the numerical solutions for the displacement, temperature and stress. Numerical results for the temperature distribution, displacement and thermal stress are represented graphically. A comparison is made with the results predicted by the three theories.

Keywords: Finite Element Method, Dual-Phase-Lag Model, Lord-Shulman Theory.

1. INTRODUCTION

The generalized theories of thermoelasticity, which admit the finite speed of thermal signal, have been the center of interest of active research during last three decades. These theories remove the paradox of infinite speed of heat propagation inherent in the conventional coupled dynamical theory of thermoelasticity introduced by Biot.[1] The theory of couple thermoelasticity was extended by Lord and Shulman (LS)[2] and Green and Lindsay[3] by including the thermal relaxation time in constitutive relations. The theory was extended for anisotropic body by Dhaliwal and Sherief.[4] Tzou[5,6] proposed the dual-phase-lag (DPL) model, which describes the interactions between phonons and electrons on the microscopic level as retarding sources causing a delayed response on the macroscopic scale. For macroscopic formulation, it would be convenient to use the (DPL) model for investigation of the micro-structural effect on the behavior of heat transfer. The physical meanings and the applicability of the (DPL) model have been supported by the experimental results.[7] The dual-phase-lag (DPL) proposed by Tzou[7] is such a modification of the classical thermoelastic model in which the Fourier law is replaced by an approximation to a modified Fourier law with two different time translations: a phase-lag of the heat flux t_q and a phase-lag of temperature gradient t_θ. A Taylor series approximation of the modified Fourier law, together with the remaining field equations leads to a complete system of equations describing a dual-phase-lag thermoelastic model. Abouelregeal[8] studied a problem of a semi-infinite medium subjected to exponential heating using a dual-phase-lag thermoelastic model. During the last three decades a number of investigations[9–12] have been carried out using the aforesaid theories of generalized thermoelasticity. Chandrasekharaiah[13] studied one-dimensional waves in a homogeneous isotropic half-space due to sudden inputs of temperature and stress/strain on the boundary by employing the Laplace transform method in the context of thermoelasticity without energy dissipation. Note that in most of the earlier studies, mechanical or thermal loading on the bounding surface was considered to be in the form of a shock. It is thus felt that a finite rise time of external load (mechanical or thermal) applied on the surface should be considered while studying a practical problem of this nature. Considering this aspect of rise time, Misra et al.[14,15] and Youssef[16] solved some problems involving a ramp-type heating at the bounding surface. Abbas and Abbas et al.[17–20] applied the finite element method in different problems. Recently Refs. [21–23], studied other problems in waves.

The present investigation is devoted to study the thermoelastic interactions in a semi-infinite medium subjected to a ramp-type heating using the finite element method (FEM). Numerical results for the temperature distribution, displacement and thermal stress are represented graphically. Finally, the comparisons are made between the

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results predicted by the coupled theory (CT), Lord and Shulman theory (LS) and dual-phase-lag model (DPL).

2. BASIC EQUATION AND FORMULATION

For a linear, homogenous and isotropic thermoelastic continuum, the generalized field equations can be presented in a unified form as[8]

$$\rho\frac{\partial^2 u_i}{\partial t^2} = (\lambda+\mu)u_{j,ji} + \mu u_{i,jj} - \gamma T_{,i} + F_i \tag{1}$$

The equation of heat conduction under dual-phase-lag model

$$\left(1+t_\theta\frac{\partial}{\partial t}\right)KT_{,ii} = \rho c_e\left(\frac{\partial}{\partial t}+t_q\frac{\partial^2}{\partial t^2}\right) \times\left(T+\gamma T_0\frac{\partial u_{j,j}}{\partial t}-\rho Q\right) \tag{2}$$

The constitutive equations are given by

$$\tau_{ij} = \lambda u_{i,j}\delta_{ij} + \mu(u_{i,j}+u_{j,i}) - \gamma(T-T_0)\delta_{ij} \tag{3}$$

Three cases arise:

(i) Classical dynamical coupled theory (CT)

$$t_\theta = t_q = 0$$

(ii) Lord and Shulman's theory (LS)

$$t_\theta = \tau > 0, \quad t_q = 0$$

(iii) Dual phase model (DPL)

$$0 < t_\theta < t_q$$

Let us consider a homogeneous, isotropic, thermoelastic solid, occupying the region $x \geq 0$ where the x-axis is taken perpendicular to the bounding plane of the half-space pointing inwards. It assumed that the state of the medium depends only on x and the time variable t. The medium described above is considered to be exposed to ramp-type surface heating described mathematically as

$$T|_{x=0} = \begin{cases} 0 & t \leq 0 \\ T_1\dfrac{t}{t_0} & 0 < t \leq t_0 \\ T_1 & t > t_0 \end{cases} \tag{4}$$

T_1 being a constant. It is assumed that there are no body forces and heat sources in the medium and that the plane $x = 0$ is taken to be traction free. Thus the field Eqs. (1)–(3) in a one-dimensional case can be put as

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2} \tag{5}$$

$$\left(1+t_\theta\frac{\partial^2}{\partial t^2}\right)\frac{\partial^2 T}{\partial x^2} = \left(\frac{\partial}{\partial t}+t_q\frac{\partial^2}{\partial t^2}\right)\left(\frac{\rho c_e}{K}T+\frac{\gamma T_0}{K}\frac{\partial u}{\partial x}\right) \tag{6}$$

$$\tau_{xx} = (\lambda+2\mu)\frac{\partial u}{\partial x} - \gamma(T-T_0) \tag{7}$$

Write $c^2 = (\lambda+2\mu)/\rho$ and $\chi = K/\rho c_e$ for convenience, we shall use the following nondimensional variables:

$$(x^\circ, u^\circ) = \frac{c}{\chi}(x, u), \quad (T^\circ, T_1^\circ) = \frac{1}{T_0}(T-T_0, T_1)$$

$$(t^\circ, t_\theta^\circ, t_q^\circ, t_0^\circ) = \frac{c^2}{\chi}(t, t_\theta, t_q, t_0), \quad \tau_{xx}^\circ = \frac{\tau_{xx}}{\mu}$$

Into Eqs. (5)–(7), one may obtain (after dropping the superscript ° for convenience)

$$\frac{\partial^2 u}{\partial x^2} - \beta_1\frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2} \tag{8}$$

$$\left(1+t_\theta\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial x^2} = \left(\frac{\partial}{\partial t}+t_q\frac{\partial^2}{\partial t^2}\right)\left(T+\beta_2\frac{\partial u}{\partial x}\right) \tag{9}$$

$$\tau_{xx} = a_1\frac{\partial u}{\partial x} - a_2 T \tag{10}$$

where $a_1 = \rho c^2/\mu$, $a_2 = T_0\gamma/\mu$, $\beta_1 = T_0\gamma/\rho c^2$, $\beta_2 = \gamma/\rho c_e$, in which $\gamma = (3\lambda+2\mu)\alpha_t$. The nondimensional forms of the initial and boundary condition are:

$$u(x,0) = \frac{\partial u(x,0)}{\partial t} = 0, \quad T(x,0) = \frac{\partial T(x,0)}{\partial t} = 0 \tag{11}$$

$$\tau_{xx}(0,t) = 0, T(0,t) = \begin{cases} 0 & t \leq 0 \\ T_1\dfrac{t}{t_0} & 0 < t \leq t_0 \\ T_1 & t > t_0 \end{cases} \tag{12}$$

3. FINITE ELEMENT METHOD

In order to investigate the thermoelastic interactions in an elastic half space subjected to a ramp-type heating, the (FEM)[24] is adopted due to its flexibility in modeling layered structures and its capability in obtaining full field numerical solution. The governing Eqs. (8) and (9) are coupled with initial and boundary conditions (11) and (12). The numerical values of the dependent variables like displacement u and the temperature T are obtained at the interesting points which are called degrees of freedom. The weak formulations of the non-dimensional governing equations are derived. The set of independent test functions to consist of the displacement δu and the temperature δT is prescribed. The governing equations are multiplied by independent weighting functions and then are integrated over the spatial domain with the boundary. Applying integration by parts and making use of the divergence theorem reduce the order of the spatial derivatives and allows for the application of the boundary conditions. The same shape functions are defined piecewise on the elements. Using the Galerkin procedure, the unknown fields u and T and the corresponding weighting functions are approximated by the same shape functions. The last step towards the finite element discretization is to choose the element type and the associated shape functions. On the other hand, the time derivatives of the unknown variables have to be determined by Newmark time integration method or other methods.[24]

In particular, the equation of motion become

$$\int_\Gamma \frac{\partial \delta u}{\partial x}\left[\frac{\partial u}{\partial x}-\beta_1 T\right]dx+\int_\Gamma \delta u\frac{\partial^2 u}{\partial t^2}\,dx = \delta u\left[\frac{\partial u}{\partial x}-\beta_1 T\right]_\Gamma \tag{13}$$

The energy equation has the form

$$\int_\Gamma \frac{\partial \delta T}{\partial x}\left(1+t_\theta\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}\,dx + \int_\Gamma \delta T\left(\frac{\partial}{\partial t}+t_q\frac{\partial^2}{\partial t^2}\right)\left(T+\beta_2\frac{\partial u}{\partial x}\right)dx = \delta T\left[\left(1+t_\theta\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}\right]_\Gamma \tag{14}$$

4. NUMERICAL RESULTS AND DISCUSSION

In order to illustrate the problem, the copper material was chosen for purposes of numerical evaluations. The physical data which given as[8]

$$\lambda = 7.76\times 10^{10}\ (\mathrm{kg})\ (\mathrm{m})^{-1}\ (\mathrm{s})^{-2}$$
$$\mu = 3.86\times 10^{10}\ (\mathrm{kg})\ (\mathrm{m})^{-1}\ (\mathrm{s})^{-2}$$
$$T_0 = 293\ (\mathrm{K}),\quad K = 3.68\times 10^{2}\ (\mathrm{kg})\ (\mathrm{m})\ (\mathrm{K})^{-1}\ (\mathrm{s})^{-3}$$
$$c_e = 3.831\times 10^{2}\ (\mathrm{m})^{2}\ (\mathrm{K})^{-1}\ (\mathrm{s})^{-2}$$
$$\rho = 8.954\times 10^{3}\ (\mathrm{kg})\ (\mathrm{m})^{-3},\quad \alpha_t = 17.8\times 10^{-6}\ (\mathrm{K})^{-1}$$

The field quantities, temperature, displacement and stress depend not only on the time t and space x, but also depend on rise-time parameter t_0 and on dual-phase-lag parameters t_θ, t_q. The results for displacement, temperature and stress has been carried out by taking $T_1 = 1$. Figures 1–3 exhibit the variation of the displacement, temperature and stress with space x under three theories. The solid line (—) refer to the classical dynamical coupled theory CT ($t_\theta = t_q = 0$). The dashed line (- - -) refer to the Lord and Shulman theory LS ($t_\theta = 0, t_q = \tau = 0.05$). The dot line (…) refer to the dual-phase-lag model DPL ($t_\theta = 0.02, t_q = 0.05$). Figures 4–6 show that the effect of rise-time parameter t_0 under dual-phase-lag model (DPL). It is obvious from Figures 1 and 4 that the displacement is negative at $x = 0$ where its magnitude is maximum. The displacement increases from the negative value to a positive value. In the positive values, the displacement has a peak value that depends on the values of the rise-time and the theories CT, LS and DPL. It is obvious from Figures 2 and 5 that the temperature decreases with the increase of the space but they increase when decreasing the rise-time parameter. There is significant difference in the value of temperature is noticed for the CT, LS and DPL theories. It is obvious from Figures 3 and 6 which give the stress variation at different instants of rise-time and three theories with the space. Its magnitude increases from zero to a maximum value after that decreases rapidly as x increases.

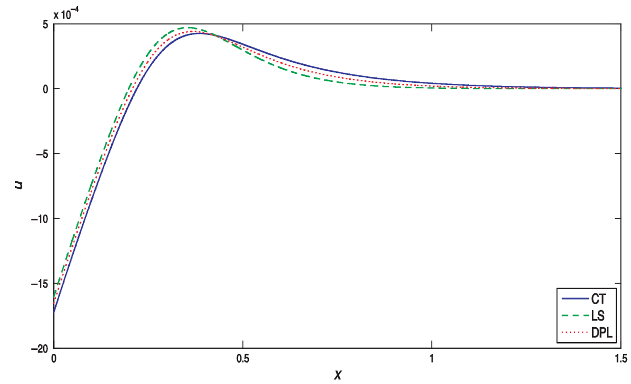

Fig. 1. Variation of displacement with distance x at time $t = 0.2$ under three theories.

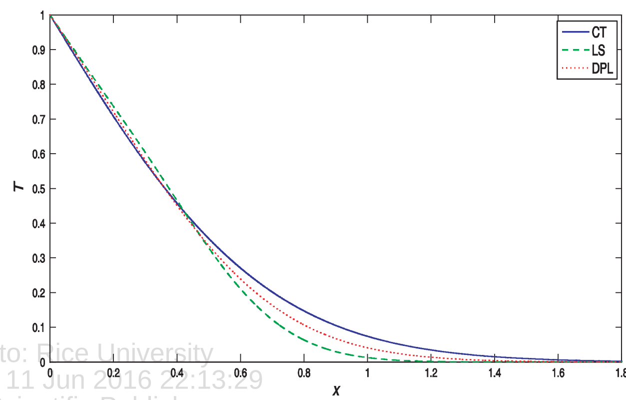

Fig. 2. Variation of temperature with distance x at time $t = 0.2$ under three theories.

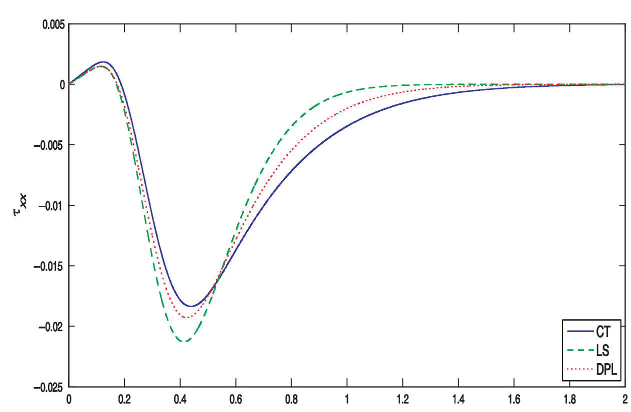

Fig. 3. Variation of stress with distance x at time $t = 0.2$ under three theories.

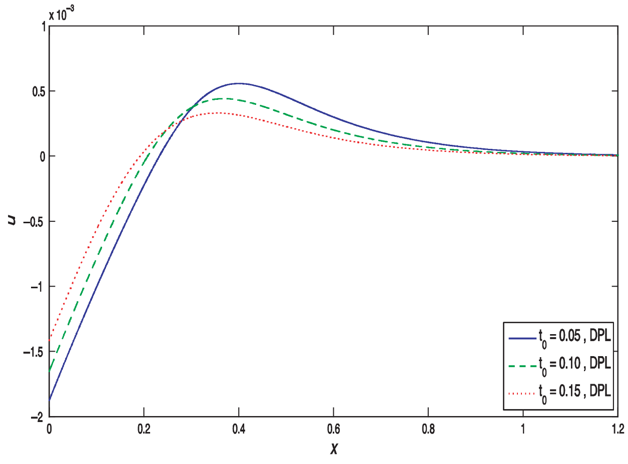

Fig. 4. Variation of displacement with distance x for different values of t_0 under DPL model at time $t = 0.2$.

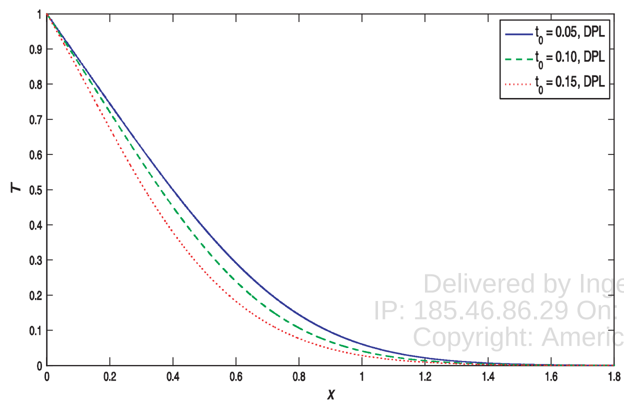

Fig. 5. Variation of temperature with distance x for different values of t_0 under DPL model at time $t = 0.2$.

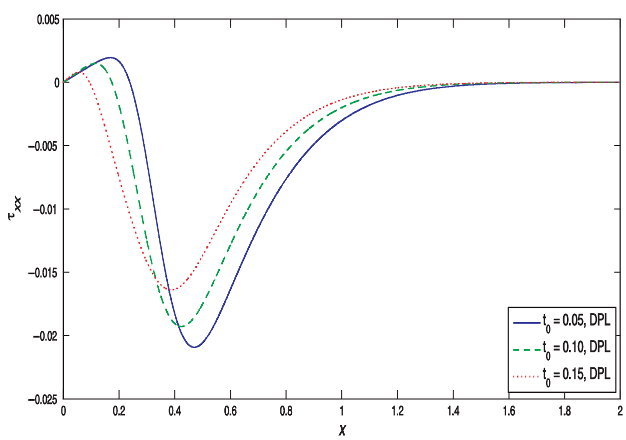

Fig. 6. Variation of stress with distance x for different values of t_0 under DPL model at time $t = 0.2$.

LIST OF SYMBOLS

λ, μ Lame's constants
ρ Density of the medium
c_e Specific heat at constant strain
α_t Coefficient of linear thermal expansion
t Time
T Temperature
T_0 Reference temperature
K Thermal conductivity
Q Heat source
t_θ Phase-lag of temperature gradient
t_q Phase-lag of the heat flux
t_0 Time of rise of temperature
δ_{ij} Kronecker symbol
Γ Domain
$\delta u, \delta T$ The weighting functions
τ_{ij} Components of stress tensor
u_i Components of displacement vector
F_i Body force vector.

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